Lock Congestion Relief in a Multimodal Network With Public Subsidies and Competitive Carriers: A Two-Stage Game Model

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This work was supported in part by the Guangxi Philosophy and Social Science Planning Research Project (21FGL025), the China Scholarship Council (202008455020), and the Ministry of Science and Technology in Taiwan (111-2410-H-194-029-MY2).

ABSTRACT Transshipment can be a detour for carriers to bypass congested locks. Therefore, the local government provides subsidies to carriers reluctant to adopt transshipment due to high costs. Using the Three Gorges Dam (TGD) as the subject, we address the interaction between the government and carriers and the rational routine choice for carriers when facing severe congestion. Specifically, we investigate pricing competition among carriers under different scenarios. A two-stage game model based on Evolutionary game theory and Bertrand game is used for the study. The results confirm that: 1) Subsidies for the road alternative can alleviate congestion in waterways transport before TGD; 2) Road transport is an efficient way to alleviate lock congestion, especially under emergency states; 3) Public subsidies for road transport support this change of modes at a reasonable price to shippers. Additionally, carriers with transshipment mode can provide more competitive freight prices and more convenient services to customers.

INDEX TERMS Subsidies, pricing, evolutionary game theory, Bertrand game.

I. INTRODUCTION

Inland waterways, such as the Rhine and Mississippi rivers, play a key role in freight transportation systems and contribute to the economic development of their regions. As a competitive alternative to road and rail transport, they are characterized by reliability, large channel capacity, low transportation cost, and environment-friendliness [1]. For example, the Yangtze River is the third-longest in the world; it contributes to the development of the economic belt in China and through the inland waterway network connecting it to the sea, goods from inland cities in China can be delivered easily to Singapore and other European countries. Since 2006, the Yangtze has surpassed the Mississippi and Rhine rivers in freight volume. However, locks, such as Three Gorges Dam (TGD), have become bottlenecks and impeded the development of Yangtze Economic Belt (Figure 1 from https://cjhy.mot.gov.cn/).

As is well known, waterway locks are indispensable to help overcome height differences between two adjacent river segments, and also bring benefits for power generation and flood prevention [2], [3]. However, they can also become...
choke points for transport when traffic volumes exceed their navigation capacity, resulting in severe time delays and economic losses to carriers, shippers and consumers. Delays can become even longer under circumstances such as equipment malfunctions, traffic accidents, extreme weather conditions, or lockage maintenance.

To relieve lock congestion, several solutions can be found in the literature, such as lock scheduling [4], [5], [6], [7], [8], the construction of new locks [9], lock capacity expansion [10], [13], [12], parallel channels [13] or the construction of ship lifts [11]. However, these solutions cannot always be applied effectively due to spatial and geographical restrictions and may create new negative environmental impacts. Furthermore, the solution may need a long-term project planned by the government which must invest billions in constructing it, maybe more than 10 years.

Other transport options, such as re-routing of freight by other modes after transshipment, have also been proposed to alleviate lock congestions (e.g., the synchronomodality roadmap in [14]). Because the cost of transshipment and re-routing usually exceeds that of the waterways option, carriers are reluctant to accept this as a solution. The government may consider monetary incentives, such as subsidies, to encourage carriers. Also, with subsidies or rewards from the government, carriers still compete for prices. Ignoring pricing competition among the carriers might lead to the risk of losing customers. Conversely, observing price competition could motivate carriers to adjust their prices to attract more market share.

There are examples of subsidies as successful price-based logistics and freight transportation instruments. For example, the Chinese government provided subsidies for energy-efficient products of two firms that compete on product prices, intending to reduce total energy consumption level [15]. Chen et al. [16] optimized government subsidies in a setting of retail and wholesale pricing decisions to increase the sustainability of a supply chain. Yang et al. [17] considered the effects of subsidies to improve clean innovation and found that more subsidies can help decrease environmental emissions. Yang et al. [1] addressed the waterway lock congestion problem earlier and explored the impact of different subsidy policies from the government on carriers’ choices.

Previous papers did not consider carrier pricing strategy and pricing competition, and questions remain about which freight price is plausible under different scenarios, given the carriers’ bounded rational behavior. In our previous research [1], we explored the impact of different subsidy policies from the government on carriers’ behavior strategies. This study extends the previous research and considers pricing strategy and pricing competition under lockage mode and transshipment mode between different carriers, which are still lacking in the available literature. Thus, we aim to fill this gap to explore which freight price is more competitive under different modes. Furthermore, the carriers’ behavioral bounded rationality is studied to avoid huge losses before locks.

In sum, we try to answer the following questions:
(1) What should be the carriers’ rational choice, between the option to wait at the lock and to transfer to another transport mode?
(2) With the public subsidies, what are the pricing strategies under different modes?
(3) Which freight price is competitive under different scenarios of lock congestion and subsidies?

Our contributions to the literature are the following:
(1) Previous research on the topic [1] was extended. To our knowledge, this is the first mathematical model to investigate the gaming strategies around bottlenecks in a multimodal network, between the authority and carriers, including price competition among carriers. We analyze carriers’ strategies under public subsidies when carriers and the public authority behave with bounded rationality. Public subsidies cause the system to evolve into an evolutionarily stable state. Price competition among carriers allows them to adjust their prices to attract market share and get the maximum profits.

(2) As an empirical contribution, we find that carriers will adopt the transshipment mode only with government subsidies. As such, the government can keep subsidies at a level that yields maximal profits for the carriers. Furthermore, we note that price competition among carriers can increase the market share and improve the service level. Interestingly, to carriers with transshipment mode, a high level of subsidy may negatively impact the optimal profit in transshipment mode.

The rest of this paper is organized as follows. Section II formulates the modelling problem and two models to determine subsidy levels (questions 1 and 2) and price competition (question 3). Then, Section III analyzes simulation results with a case study. Section IV and Section V give the discussion and the conclusion, separately.

II. MODELLING APPROACH
A. TWO-STAGE STRUCTURE

The real-world problem is as follows: given a budget of subsidies M, which subsidies should the authority provide to encourage carriers to adopt the alternative mode, to alleviate the congestion? Here we take into account price competition among carriers with different transportation modes. As shown in Figure 2, the public authority influences the use of the alternative mode and competing carriers wish to serve consumers with higher efficiency and competitive price.

The model has two stages. The first stage is based on Evolutionary Game Theory (EGT), while the second stage on the Bertrand Game. EGT provides an effective analytical tool for studying the relationship between policymakers and receptors [17]. In the EGT model, the public authority and carriers are the participants of a game under bounded rationality and uncertainty. The authority provides a strategy of subsidy or non-subsidy, with transshipment and non-transshipment as the carrier’s options. Both of them use continuous learning methods to play multiple games and seek the evolutionary
stable strategy (ESS) for achieving the optimal equilibrium with the maximal benefits. We extend this model with the Bertrand Game, to model pricing competition between the two parties [18]. Here, we consider two kinds of firms competing in prices that offer homogeneous transportation services with substitutable performance. One kind of firm is pursuing maximal profit under the lockage mode with more delay time, leading to a probability of payment for fine. Another kind of firm is to pursue maximal welfare under the lockage mode with the authority with subsidies, respectively, are:

\[ F_x = x(1 - x)\left[y(q_k s_k - C_g^e) + p_k^l q_k^l - q_k^l c_k^l - p_k^l q_k^l + q_k^l c_k^l\right] \]

\[ F_y = y(1 - y)\left[(C_g^e - q_k s_k) x - C_g^e\right] \]

The Jacobian matrix of the above replicator dynamic system is as follows:

\[
J = \begin{bmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{bmatrix}
= \begin{bmatrix}
(1 - 2x)\left[y(q_k s_k - C_g^e) + p_k^l q_k^l - q_k^l c_k^l - p_k^l q_k^l + q_k^l c_k^l\right] \\
(C_g^e - q_k s_k)(1 - 2y)[(C_g^e - q_k s_k) x - C_g^e]
\end{bmatrix}
\]

\[
\text{Det}(J) = (1 - 2x)\left[y(q_k s_k - C_g^e) + p_k^l q_k^l - q_k^l c_k^l - p_k^l q_k^l + q_k^l c_k^l\right] \\
+ p_k^l q_k^l C_g^e - p_k^l q_k^l C_g^e(1 - 2y) \\
[(C_g^e - q_k s_k) x - C_g^e] - x(1 - x)y(1 - y) \left(q_k s_k - C_g^e\right) C_g^e - q_k s_k \right)
\]

\[
\text{Tr}(J) = (1 - 2x)\left[y(q_k s_k - C_g^e) + p_k^l q_k^l - q_k^l c_k^l + q_k^l c_k^l\right] \\
+ (1 - 2y)[(C_g^e - q_k s_k) x - C_g^e] 
\]

We assume \( C_g^e < q_k s_k \), then \( x^* = C_g^e/(C_g^e - q_k s_k) \), \( y^* = \frac{\zeta}{(C_g^e - q_k s_k)} \) do not satisfy the condition of \( x^* \in [0, 1], y^* \in [0, 1] \).

Let \( \zeta = p_k^l q_k^l - q_k^l c_k^l - p_k^l q_k^l + q_k^l c_k^l \), then, Table 3 shows the different equilibrium points of the above Jacobian matrix.

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**Table 2. Payoff matrix.**

<table>
<thead>
<tr>
<th>Strategy type</th>
<th>Carrier</th>
<th>Transshipment (x)</th>
<th>Non-transshipment (1-x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Authority</td>
<td>( \Gamma_1 )</td>
<td>( \Pi_1 )</td>
<td>( \Gamma_2 \Pi_2 )</td>
</tr>
<tr>
<td>Non-subsidy</td>
<td>( \Gamma_3 )</td>
<td>( \Pi_3 )</td>
<td>( \Gamma_4 \Pi_4 )</td>
</tr>
</tbody>
</table>

**Table 3. Determinants and traces of the Jacobian matrix for different equilibrium points.**

\[
\{ \Gamma_1, \Pi_1 \} = \left\{ \pi_{k,g}^{m} - C_g^{m} - q_k s_k, p_k^l q_k^l - q_k^l c_k^l + q_k s_k \right\}
\]

\[
\{ \Gamma_2, \Pi_2 \} = \left\{ -C_g^{m} - C_g^{e} - C_g^{e}, p_k^l q_k^l - q_k^l c_k^l - C_g^{e} \right\}
\]

\[
\{ \Gamma_3, \Pi_3 \} = \left\{ \pi_{k,g}^{l} - C_g^{m} - C_g^{e}, p_k^l q_k^l - q_k^l c_k^l + q_k s_k \right\}
\]
C. PRICE SETTING MODEL

In the Bertrand model, the demand functions with subsidy under lockage mode ($D_l$) and transshipment mode ($D_t$) are:

$$D_l = \rho a - b_1 p_l + b_2 (p_l + s_l)$$

$$D_t = (1 - \rho)a - b_1 (p_t + s_l) + b_2 p_t$$

where $a$ represents the basic market demand and $\rho$ represents the carriers’ loyalty to transshipment mode with $0 \leq \rho \leq 1$.

The parameters $b_1$ and $b_2$ measure the price coefficient of demand to freight price $p_l$ of lockage mode and $p_t$ of transshipment mode sensitivity, respectively. The profit functions under lockage mode and transshipment mode (with subsidies) are:

$$\pi_l = (p_l - c_l)D_l = (p_l - c_l)[\rho a - b_1 p_l + b_2(p_l + s_l)]$$

$$\pi_t = (p_t - c_t)D_t = (p_t - c_t)[(1 - \rho) a - b_1(p_t + s_l) + b_2p_t]$$

**Theorem 1:** In the Bertrand game, when $2b_2 > b_1$, the equilibrium freight price, profit, and freight volume under different modes (10)–(15), as shown at the bottom of the page.

**Proof:** See Appendix A.

**Corollary 1:** For the optimal prices of carriers under different modes, in the Bertrand game, we have $\frac{\partial \pi_l}{\partial \rho} > 0$, $\frac{\partial \pi_l}{\partial b_1} < 0$, $\frac{\partial \pi_l}{\partial b_2} > 0$; $\frac{\partial \pi_t}{\partial \rho} < 0$, $\frac{\partial \pi_t}{\partial b_1} < 0$, $\frac{\partial \pi_t}{\partial b_2} > 0$, $\frac{\partial \pi_t}{\partial s_l} < 0$.

**Proof:** See Appendix B.

Taking the first-order derivative of $p_l^*$ and $p_t^*$ with respect to these parameters $\rho$, $s_l$, $b_1$ and $b_2$, it shows that the increase of $\rho$, $s_l$ and $b_2$ will raise the freight price of lockage mode, but $b_1$ will lower the price of lockage mode. The increase of $b_2$ will raise the freight price of transshipment mode, while the increase of $\rho$, $s_l$ and $b_1$ will reduce the price of transshipment mode. This means that the increase of $b_2$ will lower the profits of both lockage and transshipment modes, while the increase of $b_2$ will improve the profits of both modes.

**Corollary 2:** For the optimal profits of carriers under different modes, in the Bertrand game, we have $\frac{\partial \pi_l^*}{\partial \rho} < 0$, $\frac{\partial \pi_l^*}{\partial b_2} > 0$, $\frac{\partial \pi_t^*}{\partial b_1} > 0$.

**Proof:** See Appendix C.

III. APPLICATION

A. CASE DESCRIPTION

We demonstrate the working of the model for the case of the Three Gorges Dam in the Yangtze River. In 2021, the river’s throughput exceeded the design capacity of the TGD locks by more than 48%, leading to serious congestion and pollution. According to statistics released by the Three Gorges Navigation Authority, the average waiting period for ships...
TABLE 4. Initial parameter values.

<table>
<thead>
<tr>
<th>$q_x$</th>
<th>$p_x$</th>
<th>$x_i$</th>
<th>$z_i$</th>
<th>$x'_i$</th>
<th>$z'_i$</th>
<th>$c_x$</th>
<th>$c'_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>700</td>
<td>1</td>
<td>1.05</td>
<td>0.45</td>
<td>10000</td>
<td>300000</td>
<td>55000</td>
</tr>
</tbody>
</table>

The Three Gorges Transshipment System was designed by the Gorges Navigation Authority to divert ships and thus alleviate congestion, with a deadline of 2035. Today, the South Transshipment Highway is already in operation, facilitating container and Roll-on/Roll-off (RO-RO) ships to transship goods to the road mode and back (see Figure 3).

Cargoes can be shipped from the Zigui Terminal or Baiyang Terminal to their destination by water-land mode. In 2018, the ratio of container to RO-RO ships adopting the water-land transshipment strategy was 2%, far below the authority’s expectations. As transshipment incurs additional transit costs, it is more expensive than waterway transportation. This has become an obstacle for carriers to adopt transshipment mode. Thus, since 2019, the authority has endeavored to increase the transshipment freight volume and mitigate congestion pressure by providing a two-year 50% discount on differential tolls for transshipment trucks to pass through the South Transshipment Highway. Furthermore, every transshipment carrier can receive a 3-year subsidy from the local authority.

Changan Minsheng APLL Logistics Co. Ltd. (CNAL) and China Changjiang National Shipping Group Co. Ltd. (CCNSC) are both engaged in vehicle transportation for car manufacturers from Chongqing to Wuhan. CNAL is a private company pursuing profit maximization and CCNSC is a state-controlled firm whose ownership is endogenously chosen by a welfare-maximizing authority, with pricing competition between the two enterprises. From 1st October 2019 onwards, the Three Gorges Navigation Authority has canceled priorities for all ships, and they must queue for passing locks.

The initial values in the evolutionary game model are listed in Table 4.

B. RESULTS

With the main parameters in Table 5 and initial values $(0.5, 0.53)$ for $(x, y)$, when $\zeta < 0$, $\zeta + q_x k_x < 0$ satisfies the current state. We can see in Figure 5 that the initial evolutionary stable strategy, $ESS(0,0)$, is the steady-state when all the evolutionary curves converge with changes of $t$ in $(0,1]$. This shows that the best strategy to maximize benefits for carriers is non-transshipment and for the authority is non-subsidy. Thus, Scenario 2 is verified.

As Figure 4 shows, carriers can get the maximal benefits without adoption of transshipment mode, although it is against the authority’s expectation. With the aim to divert the flow of ships, to alleviate the congestion pressure for the TGD, and to improve traffic efficiency, the authority has provided subsidies as the incentives to the carriers. With the
subsidies, carriers can provide customers with a better level of service.

To better analyze our theoretical results, in the Bertrand game, let $b_1 = 20$, $b_2 = 20$, $a = 1$, $s_I = 1.5$, the waiting cost be 0.52 RMB/vehicle, and the waiting time be 3 days. The results of the analyses are presented in Table 6, and Figures 6 to 8. Each figure represents the output of one modelling and simulation experiment.

The model provides interesting insights into the effect of several parameters on optimal prices and profits, as similar in other studies.

**Sensitivity Analysis:** In order to validate models, we test the sensitivity of the following parameters. In addition, we also want to find the effect of key parameters on price and profits for carriers.

- loyalty parameter (Table 6)
- subsidies (Figure 5)
- price coefficients (Figures 6-7)
- lock capacity and subsidy (Figure 8)

**Effect of Loyalty Parameter:** From Table 6, we can see prices are not very sensitive to changes in $\rho$. The direction of the response is as expected. Under *without subsidies and congestion* scenario, shippers will pay more with transshipment mode with $p_I^*$ than lock mode due to high transshipment cost. In addition, the freight price $p_I^*$ in the road mode is less competitive than $p_I^*$ under the lockage mode. Under other scenarios, the freight price $p_I^*$ of the road mode is lower and more competitive than that of waterways. This facilitates carriers to transfer to transshipment mode while providing a more reasonable freight price and more convenient service.

Profits are more sensitive to the loyalty parameter. Optimal profits $\pi_I^*$ and $\pi_t^*$ all show different changes with the parameter $\rho$ as shown in Table 6 too. Only when no subsidies are available and there is congestion, the increase of $\rho$ will lower $\pi_I^*$ and $\pi_t^*$, and carriers in lockage mode will earn fewer profits than carriers in transshipment mode because of congestion. Similarly, when there are neither subsidies nor congestion, the lockage mode is the best choice for carriers to achieve maximal profits. But when subsidies are available, no matter whether congestion exists or not, carriers in transshipment mode will earn lower profits than carriers in lockage mode because the public subsidies are not enough to offset the additional cost due to transshipment service.

Additionally, from Table 6, only under *without subsidies, with congestion* scenario, can the changes of demand on lockage loyalty seem reasonable. This is due to the severe congestion before TGD, carriers should rationalize to take transshipment as a detour to pass locks to save time costs. However, under others scenarios, carriers should use lockage mode as a rational behavior so as to delivery cargoes quickly.

**Effect of Subsidy Level:** The optimal prices as a function of the subsidy provided show some interesting patterns. Figures 5a and 5c show that when $\rho = 0.5$ (i.e., equal loyalty to lockage and transshipment modes), optimal price $p_I^*$ decreases with $s_I$ and optimal price $p_I^*$ increases with $s_I$, with or without congestion.

With the public subsidies, carriers choosing transshipment mode can provide a more reasonable freight price and more convenient service to shippers compared to carriers choosing lockage mode. Figure 5b illustrates that carriers can obtain maximal profit in lockage mode as the optimal profit $\pi_I^*$ is higher than $\pi_t^*$, without congestion when parameter $s_I$ increases. In Figure 5d, with congestion before the TGD, the optimal profits $\pi_I^*$ and $\pi_t^*$ decrease first and then increase with $s_I$ showing two inflection points. This implies that carriers only earn the minimal possible profits under different transshipment modes but, in practice, the waiting time changes daily and will impact the profits of the carriers. Specifically, the authority should offer other incentives such as tax reduction or refund to carriers adopting transshipment mode to reduce the profit gaps $(\pi_I^* - \pi_t^*)$ as shown in Figure 5d. One can use the formulas of $\pi_I^*$ and $\pi_t^*$ in Appendix A to find the value of the associated incentives needed for the road carriers.

**Effect of Price Coefficient Assumptions:** Figure 6-8 portray the influence of the price coefficient assumptions.

When $\rho = 0.5$, $b_2 = 15$, we can see that the optimal freight prices of $p_I^*$ and $p_t^*$ all decrease with increasing $b_1$. When subsidies are available, the optimal freight price of transshipment mode with or without congestion is lower than that of lockage mode, which shows that the freight price of transshipment mode is lower and more competitive than that of lockage mode. When no subsidies are available, the optimal freight price of lockage mode without congestion is more competitive than that of transshipment mode due to high transit cost. With severe congestion before the TGD,
It is interesting that under the scenarios with subsidies and without or with congestion, the profit for carriers in transshipment mode is less than that in lockage mode, which illustrates that the public subsidies alone are insufficient to offset the loss induced by transshipment.

Figure 8 indicates the optimal profits generated from capacity \( a \) and subsidy \( s_l \), given \( \rho = 0.5, b_1 = 15, b_2 = 15 \). When \( a \) and \( s_l \) increase, the optimal profit \( \pi^*_1 \) also increases as shown in Figures 8a and 8b. The optimal profit with congestion is much lower than that without congestion. In the transshipment mode, without congestion, Figure 8c shows that \( s_l \) increases with the optimal profit \( \pi^*_1 \), while a decrease with \( \pi^*_1 \). With congestion in Figure 8d, carriers receiving smaller subsidies can increase profits by increasing transshipment capacity. Conversely, while carriers receiving larger subsidies, the optimal profit \( \pi^*_1 \) decreases rapidly with increasing transshipment capacity. This illustrates how a high level of subsidy negatively impacts profits for the alternative mode.

**IV. DISCUSSION**

Considering severe congestion before locks, we find that the carriers’ rational behavior to bypass congested locks in a short time can help extend the extent research, especially with the public subsidies provided to carriers who adopt transshipment mode. Based on this, we constructed a two-stage game model to analyze the interaction between the government and carriers, as well as price competition among heterogeneous carriers under emergency and non-emergency scenarios. When shippers are loyal to lockage mode without subsidies and congestion, the transshipment price is higher and less competitive. Under no subsidies, with congestion scenario, carriers can earn more profits with transshipment which is beneficial for carriers. With congestion before locks, when the subsidy level is increasing, the optimal profits under lockage and transshipment decrease first and then increase second. Under no subsidies and congestion state, the transshipment price is higher than the waterway price with less competitiveness with two different price coefficient parameters. Similarly, carriers with waterway mode can earn more when is set as more than 15.2 under without subsidies and congestion scenario. With price coefficient \( b_2 \), only under without subsidies, with congestion state, carriers behave rationally to choose transshipment to get maximal benefits. Furthermore, competition stimulates carriers transferring to transshipment mode to provide reasonable freight prices.

**V. CONCLUSION**

The local authority of TGD has been providing subsidies to encourage carriers to adopt transshipment mode to alleviate congestion before the dam. We constructed a two-stage game model to explore the behavioral strategies of both the authority and the carriers, as well as the possible scenarios of pricing competition between carriers adopting transshipment or lockage mode. Simulation models for different scenarios were developed to demonstrate and analyze optimal prices and profits concerning various parameters.
The study has several managerial implications. Firstly, subsidies allow carriers to reduce their prices and gain a higher profit. Second, the model indicates the minimum subsidy levels for the alternative route to be adopted. For the authority, other incentive mechanisms, such as highway toll reduction, tax benefits, or even subsidy to shippers, could be used to alleviate the congestion. Secondly, this study also offers the authority a formula to identify the profit gaps and determine the incentive mechanisms to fill up the gaps when the cost of incentive is within the authority budget. Thirdly, road carriers cannot offer more competitive pricing than the waterways carriers without public subsidies or congestion. When congestion exists, the freight price of the transshipment mode can be lower and more competitive than that of waterways, which can guide carriers to transfer. Nevertheless, if the transshipment-mode carriers receive public subsidies when there is no congestion, the optimal profit descends as the price coefficient of demand increases. Importantly, this implies that carriers will be incentivized to only adopt the alternative mode under the congestion condition, when public subsidies

$$p_t^* = \frac{2b_1\rho a + b_2(1 - \rho)a + s_t b_1 b_2 + b_1 b_2 c_t + 2b_t^2 c_t}{(2b_1 - b_2)(2b_1 + b_2)}$$

$$\frac{\partial p_t^*}{\partial \rho} = \frac{(2b_1 - b_2)a}{(2b_1 - b_2)(2b_1 + b_2)} = \frac{a}{2b_1 + b_2}$$

$$\frac{\partial p_t^*}{\partial b_t} = \frac{(2\rho a + s_t b_2 + b_2 c_t + 4b_1 c_t)(2b_1 - b_2)(2b_1 + b_2)}{(2b_1 - b_2)^2(2b_1 + b_2)^2} - \frac{[2b_1\rho a + b_2(1 - \rho)a + s_t b_1 b_2 + b_1 b_2 c_t + 2b_t^2 c_t](8b_t)}{(2b_1 - b_2)^2(2b_1 + b_2)^2} = \frac{-2\rho a b_2^2 - 8b_1 b_2(1 - \rho)a - s_t b_2(4b_1^2 + b_2^2) - c_t b_2(4b_1^2 + b_2^2) - 4b_1 b_2^2 c_t}{(2b_1 - b_2)^2(2b_1 + b_2)^2}$$

$$\frac{\partial p_t^*}{\partial b_2} = \frac{[(1 - \rho)a + s_t b_1 + b_1 c_t](2b_1 - b_2)(2b_1 + b_2)}{(2b_1 - b_2)^2(2b_1 + b_2)^2} - \frac{[2b_1\rho a + b_2(1 - \rho)a + s_t b_1 b_2 + b_1 b_2 c_t + 2b_t^2 c_t](-2b_2)}{(2b_1 - b_2)^2(2b_1 + b_2)^2} = \frac{4\rho a b_1 b_2 + (1 - \rho)a(4b_1^2 + b_2^2) + s_t b_1(4b_1^2 + b_2^2) + c_t b_1(4b_1^2 + b_2^2) + 8b_2^2 c_t}{(2b_1 - b_2)^2(2b_1 + b_2)^2}$$

$$\frac{\partial p_t^*}{\partial s_t} = \frac{b_1 b_2}{(2b_1 - b_2)(2b_1 + b_2)}$$

$$\frac{\partial p_t^*}{\partial b_t} = \frac{2b_1(1 - \rho)a + b_2\rho a + s_t(-2b_1^2 + b_2^2) + 2b_t^2 c_t + b_1 b_2 c_t}{(2b_1 - b_2)(2b_1 + b_2)}$$

$$\frac{\partial p_t^*}{\partial \rho} = \frac{-2b_1 a + b_2 a}{(2b_1 - b_2)(2b_1 + b_2)} = \frac{-a}{2b_1 + b_2}$$

$$\frac{\partial p_t^*}{\partial b_1} = \frac{[2(1 - \rho)a - 4b_1 s_t + 4b_1 c_t + b_2 c_t](4b_1^2 - b_2^2)}{(2b_1 - b_2)^2(2b_1 + b_2)^2} - \frac{[2b_1(1 - \rho)a + b_2\rho a + s_t(-2b_1^2 + b_2^2) + 2b_t^2 c_t + b_1 b_2 c_t](8b_1)}{(2b_1 - b_2)^2(2b_1 + b_2)^2} = \frac{2(1 - \rho)a(-4b_1^2 - b_2^2) + s_t(-4b_1 b_2^2) + c_t(-b_2^2) + b_2 c_t(-4b_1^2 - b_2^2)}{(2b_1 - b_2)^2(2b_1 + b_2)^2}$$

$$\frac{\partial p_t^*}{\partial b_2} = \frac{[\rho a + 2b_2 s_t + b_1 c_t](4b_1^2 - b_2^2)}{(2b_1 - b_2)^2(2b_1 + b_2)^2} - \frac{[2b_1(1 - \rho)a + b_2\rho a + s_t(-2b_1^2 + b_2^2) + 2b_t^2 c_t + b_1 b_2 c_t](-2b_2)}{(2b_1 - b_2)^2(2b_1 + b_2)^2} = \frac{\rho a(4b_1^2 + b_2^2) + 4b_1 b_2(1 - \rho)a + s_t(4b_1^2 b_2) + c_t(4b_1^2 b_2) + c_t(4b_1^3 + b_1 b_2^2)}{(2b_1 - b_2)^2(2b_1 + b_2)^2}$$

$$\frac{\partial p_t^*}{\partial s_t} = \frac{-2b_1^2 + b_2^2}{(2b_1 - b_2)(2b_1 + b_2)}$$
are provided. Finally, this study reveals that public subsidies have opposing effects on carriers’ optimal profits. With congestion, carriers receiving smaller subsidies can increase optimal profits by increasing transshipment capacity. However, under larger subsidy levels, optimal profits decrease rapidly when transshipment capacity increases. This suggests that the public authority may need to keep the subsidy at a level that yields sufficient profits for carriers.

We see several opportunities for further research.

● Within the framework of the Three Gorges Transshipment Project, the South Transshipment Railway is currently under construction. Shortly, RO-RO and container carriers will also have access to railway transshipment transportation. Research into carriers’ adoption strategy of railway transshipment would be worthwhile. Although the principles proposed here will apply similarly to rail transport, further competition will influence the results. One should consider including heterogeneous service requirements in the model to differentiate better between road and rail service options.

● Moreover, no comparison is made with other decision models, which limits confidence in the validity of the results. Future investigations into model comparisons are highly recommended to confirm the robustness and validity of the results.

● Finally, under some emergency states such as locks malfunctions, traffic accidents, and extreme weather conditions, carriers may select different transportation modes, and the behavioral strategies of mode adoption for carriers may be different from those under regular states. The adoption decision will be determined by the transportation network

\[
\pi^*_i = (p_i^* - c_i)D_i = (p_i^* - c_i)[(1 - \rho)a - b_1(p_i^* + s_i) + b_2p_i^*]
\]

\[
= \frac{2b_1\rho a + b_2(1 - \rho)a + s_i b_1 b_2 + b_1 b_2 c_i + 2b_1^2 c_i}{(2b_1 - b_2)(2b_1 + b_2)} - c_i
\]

\[
× [(1 - \rho)a - b_1(\frac{2b_1(1 - \rho)a + b_2(1 - \rho)a + s_i(-2b_1^2 + b_2^2) + 2b_1^2 c_i + b_1 b_2 c_i}{(2b_1 - b_2)(2b_1 + b_2)} + s_i)]
\]

\[
+ b_2(\frac{2b_1(1 - \rho)a + b_2(1 - \rho)a + s_i b_1 b_2 + b_1 b_2 c_i + (b_2^2 - 2b_1^2) c_i}{(2b_1 - b_2)^2(2b_1 + b_2)} + s_i)]
\]

\[
= \frac{2b_1(1 - \rho)a + b_2(1 - \rho)a + s_i b_1 b_2 + b_1 b_2 c_i + (b_2^2 - 2b_1^2) c_i}{(2b_1 - b_2)^2(2b_1 + b_2)}
\]

\[
× [(2b_1^2 + b_2^2)(s_i + c_i) + (b_1^2 b_2 - 2b_1^2) c_i]
\]

\[
\frac{\partial \pi^*_i}{\partial \rho} = \frac{2b_1 a - b_2 a}{(2b_1 - b_2)^2(2b_1 + b_2)}
\]

\[
+ \frac{2b_1 a + b_2(1 - \rho)a + b_1 b_2(1 - \rho)a + b_1^2 b_2(s_i + c_i) + (b_1 b_2^2 - b_1^3) c_i}[2b_1^2 a - b_1 b_2 a]
\]

\[
= \frac{(2b_1 - b_2) a \cdot [2b_1^2 + b_2^2] c_i + 2b_1^2 a + b_1 b_2(1 - \rho)a + b_1^2 b_2(s_i + c_i) + (b_1 b_2^2 - b_1^3) c_i][2b_1^2 a - b_1 b_2 a]}{(2b_1 - b_2)^2(2b_1 + b_2)^2}
\]

\[
+ \frac{(2b_1 - b_2) a \cdot [(b_1 b_2^2 - b_1^3) c_i + 2b_1^2 a + b_1 b_2(1 - \rho)a + b_1^2 b_2(s_i + c_i) + (b_1 b_2^2 - b_1^3) c_i][2b_1^2 a - b_1 b_2 a]}{(2b_1 - b_2)^2(2b_1 + b_2)^2}
\]

\[
= \frac{2a[2b_1^2 a + b_1 b_2(1 - \rho)a + b_1^2 b_2(s_i + c_i) + (b_1 b_2^2 - b_1^3) c_i]}{(2b_1 - b_2)^2(2b_1 + b_2)^2}
\]
equilibrium, optimality, algorithms, mode adoption strategy, and interactions between shippers and carriers. More research into these determinants is worth pursuing in the future.

**APPENDIX**

**A. PROOF OF THEOREM 1**

The profit functions are expressed as

\[ \pi_l = (p_l - c_l)D_l = (p_l - c_l)[\rho a - b_1 p_l + b_2(p_l + s_l)] \]

\[ \pi_t = (p_t - c_t)D_t = (p_t - c_t)(1 - \rho) a - b_1 (p_l + s_t) + b_2 p_t \]

The first order condition must be satisfied to achieve the equilibrium, i.e., \( \frac{\partial \pi_l}{\partial p_l} = 0, \frac{\partial \pi_l}{\partial p_t} = 0 \), if \( \frac{\partial \pi_t}{\partial p_l} = b_2(p_t - c_t) = 0 \) and \( \frac{\partial \pi_t}{\partial p_t} = b_2(p_t - c_t) = 0 \), then leading to \( p_l = c_l \) and \( p_t = c_t \), this doesn’t satisfy the profit theory, accordingly,

\[ \rho a - b_1 p_l + b_2(p_l + s_t) + (p_l - c_l)(-b_1) = 0 \]

Combining the above two formulations and get

\[ p_l^* = \frac{2b_1 \rho a + b_2(1 - \rho) a + s_t b_1 b_2 + b_1 b_2 c_l + 2b_1^2 c_l}{(2b_1 - b_2)(2b_1 + b_2)} \]

\[ p_t^* = \frac{2b_1(1 - \rho) a + b_2 p_l + s_t b_2(1 - b_2)^2 + 2b_1^2 c_l + b_1 b_2 c_l}{(2b_1 - b_2)(2b_1 + b_2)} \]

Substituting \( p_l^* \) and \( p_t^* \) to obtain the optimal demand, i.e.,

\[ D_l^* = \rho a - b_1 p_l^* + b_2(s_t + c_t) \]

\[ = \frac{2b_1^2 \rho a + b_1 b_2(1 - \rho) a + b^2_1 b_2(s_t + c_t) + (b_1 b_2^2 - 2b_1^2) c_l}{(2b_1 - b_2)(2b_1 + b_2)} \]

\[ D_t^* = (1 - \rho) a - b_1 p_t^* + b_2 p_l^* \]

\[ = \frac{2b_1^2(1 - \rho) a + b_1 b_2 p_l^* + b^2_1 b_2 c_l + b_1(b_2^2 - 2b_1^2)(s_t + c_t)}{(2b_1 - b_2)(2b_1 + b_2)} \]
\[
\frac{\partial \pi^*_1}{\partial s_t} = \frac{(b_1b_2)[2b_1^2\rho a + b_1b_2(1 - \rho)a + b_1^2b_2(s_t + c_t) + (b_1b_2^2 - 2b_1^2)c_t]}{(2b_1 - b_2)^2(2b_1 + b_2)^2} + \frac{[2b_1\rho a + b_2(1 - \rho)a + b_1b_2(s_t + c_t) + (b_1^2 - 2b_1^2)c_t][b_1b_2s_t]}{(2b_1 - b_2)^2(2b_1 + b_2)^2}
\]

\[
\frac{\partial \pi^*_2}{\partial \rho} = \frac{[-2b_1a + b_2a][2b_1^2(1 - \rho)\alpha + b_1b_2\rho a + b_1^2b_2^2 c_t + b_1(b_2^2 - 2b_1^2)(s_t + c_t)]}{(2b_1 - b_2)^2(2b_1 + b_2)^2} + \frac{[2b_1(1 - \rho)\alpha + b_2\rho a + s_t(-2b_1^2 + b_2^2) + (-2b_1^2 + b_2^2)c_t + b_1b_2c_t][-2b_1^2a + b_1b_2a]}{(2b_1 - b_2)^2(2b_1 + b_2)^2}
\]

\[
\frac{\partial \pi^*_3}{\partial b_1} = \frac{[2(1 - \rho)\alpha + s_t(-4b_1) + (-4b_1)c_t + b_1c_t]}{(2b_1 - b_2)^2(2b_1 + b_2)^2} \times \frac{[2b_1^2(1 - \rho)\alpha + b_1b_2\rho a + b_1^2b_2^2 c_t + b_1(b_2^2 - 2b_1^2)(s_t + c_t)]}{(2b_1 - b_2)^2(2b_1 + b_2)^2}
\]

\[
\frac{\partial \pi^*_4}{\partial b_2} = \frac{[\rho a + 2s_t(b_2) + (2b_2)c_t + b_1c_t]}{(2b_1 - b_2)^2(2b_1 + b_2)^2} \times \frac{[2b_1^2(1 - \rho)\alpha + b_1b_2\rho a + b_1^2b_2^2 c_t + b_1(b_2^2 - 2b_1^2)(s_t + c_t)]}{(2b_1 - b_2)^2(2b_1 + b_2)^2}
\]
Substituting $p_l^r$, $p_r^s$, $D_l^f$ and $D_r^f$ to obtain the optimal profit, i.e.,

$$
\pi^*_f = \frac{[2b_2(1-\rho)\alpha + b_2^2(1-\rho)s + c_l + c_r] + (b_2^2 - 2b_2^2)c_1}{(2b_1 - b_2)^2(2b_1 + b_2)^2} \\
\times \frac{[2b_2^2(1-\rho)\alpha + b_1b_2s + c_1]}{(2b_1 - b_2)^2(2b_1 + b_2)^2}.
$$

**B. PROOF OF COROLLARY 1**

As shown in the equation at the bottom of page 8.

**C. PROOF OF COROLLARY 2**

As shown in the equation at the bottom of pages 9–12.

**REFERENCES**


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